Identification of Nonlinear Elastic Bending Behavior for Cable Simulation

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EXTENDED ABSTRACT

In recent years, the demand for fast and realistic simulations of flexible slender structures such as cables and hoses has increased due to their widespread use in the automotive industry. For such slender flexible structures, Cosserat rod theory provides an efficient and geometrically exact modeling framework [1]. A linear constitutive behavior, namely a linear moment-curvature relationship, has often been used in the Cosserat rod theory for the study of the bending behavior of beams. This is sufficient for many applications, however, for more complex structures, nonlinear elastic behavior plays an important role [2]. In our recent work [3], we presented an iterative method for the forward simulation of the nonlinear elastic bending behavior of cables and formulated the corresponding inverse problem, i.e. we proposed a data-based method to identify the state-dependent bending stiffness characteristic for given measurement data. In this contribution, we introduce an alternative method to identify the bending stiffness characteristics of cables, based on the equilibrium equations for rods. Also, we discuss enhancements of the inverse problem, where pre-curvature is considered as additional optimization variable.

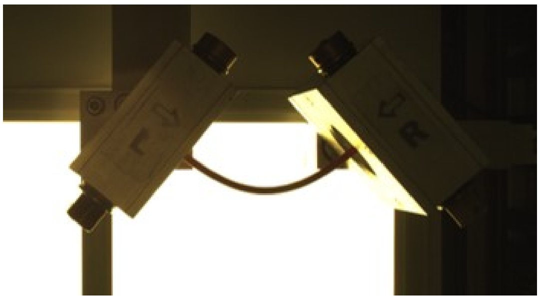

Figure 1: Top-view of MeSOMICS [4] bending experiment. The left clamping point is displaced step-wise towards the right clamping point, resulting in different bending deformations (cf. Fig. 2).

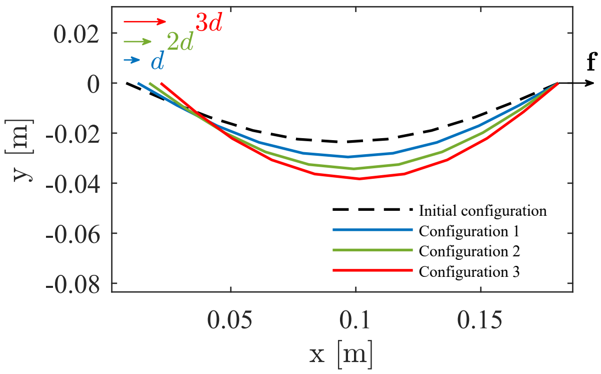

Figure 2: Generated bending configurations during the MeSOMICS bending experiment by applying displacements d, $2d$, $3d$ to the left clamping, where d is the cable diameter. On the right clamping, the resulting reaction force $\mathbf{f}$ is measured.

In absence of external body forces and moments acting on the Cosserat rod in two-dimensional space, the equilibrium equations are given by $\partial_s \mathbf{f} = 0$ and $\partial_s \mathbf{m} + \partial_s \boldsymbol{\varphi} \times \mathbf{f}(s) = 0$, where $\mathbf{f}(s) = (f^x(s), f^y(s), 0)^T \in \mathbb{R}^3$ is the force vector, $\mathbf{m}(s) = (0,0,m(s))^T \in \mathbb{R}^3$ is the moment vector, $\boldsymbol{\varphi}(s) = (x(s), y(s), 0)^T \in \mathbb{R}^3$ is the centreline and $s \in [0,L]$ represents the arc length parameter. The equilibrium equations imply that their integrals are supposed to be constant along the rod, independent of the constitutive behavior [5], and we find $\mathbf{f}(s) = \mathbf{f}$ and $\mathbf{m}(s) + \boldsymbol{\varphi}(s) \times \mathbf{f} = \boldsymbol{\mathscr{M}}$, where $\boldsymbol{\varphi}(s) \times \mathbf{f}$ and $\mathbf{m}(s)$ only have a non-vanishing z-component such that we write $m(s) + x(s) \cdot f^y - y(s) \cdot f^x = \mathscr{M}$.

More specifically, for our bending experiment [4] (see Fig. 1 and Fig. 2) with moment-free boundary conditions at both clamping points $m(0) = 0$, $m(L) = 0$, also leading to vanishing force in y-direction, i.e. $f^y = 0$, and setting the y-coordinate at the left clamping point to $y(0) = 0$, we obtain $\mathscr{M} = 0$. Thus, the bending moment at arc length s is given by $m(s) = y(s) \cdot f^x$. Moreover, the local curvature $\kappa(s)$ can be calculated as $\kappa(s) = \frac{d\theta}{ds}$, where $\theta = \arctan(\frac{dy}{dx})$. Further, the derivative of the moment with respect to the curvature

$$\left.\frac{dm(\kappa)}{d\kappa}\right|_{\kappa=\kappa(s)} =: f_{EI}\left(\kappa(s)\right) \tag{1}$$

represents the state-dependent bending stiffness for curvature $\kappa(s)$. Since our bending experiment provides an interval of curvatures (in each static configuration the curvature vanishes at the boundaries and reaches its maximum in the middle), we can determine the state-dependent bending stiffness characteristic from the graph $(\kappa(s), f_{EI}(\kappa(s)))$. Moreover, we consider several configurations as can be seen in Fig. 2 and, thus, get several graphs which can be combined.

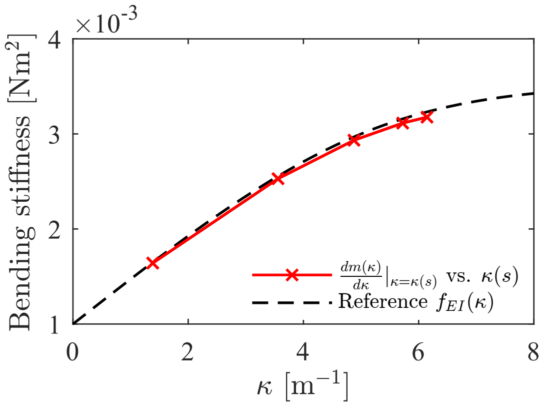

Figure 3: Used reference characteristic $f_{EI}(\kappa)$ and the identified bending stiffness characteristic by using virtual data generated with the reference characteristic.

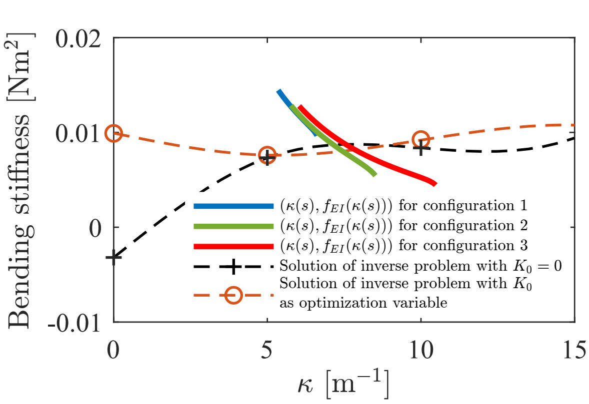

Figure 4: Identified bending stiffness characteristic using experimentally measured data. The solid lines show the $(\kappa(s), f_{EI}(\kappa(s)))$ graphs identified from (1). The dashed lines show results from the inverse problem.

First, we investigate the above described method on virtual data. With a known reference bending stiffness characteristic $f_{EI}(\kappa)$, the bending line and the reaction force are simulated by the iterative method presented in [3]. By using (1), the obtained bending stiffness characteristic agrees well with the reference characteristic, as can be seen in Fig. 3.

Second, the method is applied to experimentally measured data, where Fig. 2 shows the bending lines of all three investigated configurations. From these bending lines and the measured forces, the $(\kappa(s), f_{EI}(\kappa(s)))$ graphs are identified for all of the three configurations, which show good consistency (solid lines in Fig. 4).

Moreover, Fig. 4 is complemented by solutions of the inverse problem: on the one hand (black dashed line), with a priori defined constant pre-curvature $K_0 = 0$ and, on the other hand (orange dashed line), with constant pre-curvature as optimization variable (identified constant pre-curvature $K_0 \approx 3\,\mathrm{m}^{-1}$). As can be seen, solving the inverse problem with a priori defined pre-curvature $K_0 = 0$ leads to an unphysical characteristic with negative bending stiffness for small curvatures, while including the pre-curvature as optimization variable provides a more realistic result.

However, although both approaches – using (1) to find $(\kappa(s), f_{EI}(\kappa(s)))$ graphs and the inverse problem – provide bending stiffness characteristics in the same order of magnitude, they obviously show qualitatively different behavior. A more detailed investigation comparing both approaches is ongoing research. One aspect is to estimate the error in the computation of $\kappa(s)$ from optically detected bending lines $\varphi(s) = (x(s), y(s))^T$. Further, we aim to investigate the influence of pre-curvature K_0 also on the $(\kappa(s), f_{EI}(\kappa(s)))$ graphs.

Summarizing, in this contribution, we continue our work on simulating nonlinear elastic bending behavior of real cables. While an efficient forward simulation already could be presented in our previous work [3], now we focus our work on a robust method to identify realistic stiffness characteristics for the nonlinear elastic bending behavior of cables, which is essential for reliable simulations.

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